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CDM models with a BSI steplike primordial spectrum and a cosmological constant

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ABSTRACT

A class of spatially-flat models with cold dark matter (CDM), a cosmological constant and a broken-scale-invariant (BSI) steplike primordial (initial) spectrum of adiabatic perturbations, generated in an exactly solvable inflationary model where the inflaton potential has a rapid change of its first derivative at some point, is confronted with existing observational data on angular fluctuations of the CMB temperature, galaxy clustering and peculiar velocities of galaxies. If we locate the step in the initial spectrum at $k \simeq 0.05\,h$ Mpc^{-1}, where some feature in the spectrum of Abell clusters of galaxies was found that could reflect a property of the initial spectrum, and if the large scales flat plateau of the spectrum is normalized according to the COBE data, the only remaining parameter of the spectrum is p - the ratio of amplitudes of the metric perturbations between the small scales and large scales flat plateaux. Allowed regions in the plane of parameters ($\Omega = 1 - \Omega_\Lambda$, H_0) satisfying all data have been found for p lying in the region (0.8-1.7). Especially good agreement of the form of the present power spectrum in this model with the form of the cluster power spectrum is obtained for the *inverted* step ($p < 1$, $p = 0.7 - 0.8$), when the initial spectrum has slightly more power on small scales.

Key words: cosmology - initial spectrum of perturbations - large-scale structure of the Universe - cosmological constant.

1 INTRODUCTION

The inflationary paradigm (see the review in Linde 1990; Kolb & Turner 1990) offers an elegant solution to some of the outstanding problems of standard Big Bang cosmology. In these models, primordial quantum fluctuations (Hawking 1982, Starobinsky 1982, Guth & Pi 1982) of some scalar field(s) (inflaton(s)) are produced, which eventually form galaxies, clusters of galaxies and the large-scale structure of the Universe through gravitational instability. Though there is a variety of possible models, parametrized by a few number of constants, the increasing amount of data, for example from redshift surveys and cosmic microwave background (CMB) anisotropies measurements, severely constrain the proposed models. Hence some of them can be definitely excluded at this stage while the remaining ones are found to be viable only in some well defined region of their free parameter(s) space. Additional sharp constraints are expected from the planned satellite missions MAP and PLANCK SURVEYOR for the measurement of the CMB anisotropies up to small angular scales.

Since it is known that the simplest CDM model with a flat (n=1) initial spectrum of adiabatic perturbations does not agree with observational data (if normalized to the COBE data at large scales, it has too much power at small scales), a number of approaches to increase the ratio of large to small scale power were proposed. One possibility is to change the initial spectrum of perturbations. Since tilted scale-free spectra ($n < 1$) did not appear successful, the next step was to consider broken-scale-invariant (BSI) spectra arising in inflationary models with two effective scalar fields (Kofman, Linde & Starobinsky 1985; Kofman & Linde 1987; Silk & Turner 1987; Kofman & Pogosyan 1988; Gottlöber, Müller & Starobinsky 1991; Polarski & Starobinsky 1992). Recently, the CMB anisotropies for a model of double inflation (Lesgourgues & Polarski 1997) was investigated and it was found that for values of the parameters which yield a power spectrum $P(k)$ in fair agreement with observations,

the Doppler peak turns out to be low. This is related to the effective tilt of the spectrum on very large scales.

Another possibility is to add a positive cosmological constant leaving the initial $n \simeq 1$ spectrum unchanged. It was long known that the cosmological constant is viable only if it is accompanied by cold dark matter and, vice versa, its inclusion improves the CDM model a great deal (as emphasized e.g. in Kofman & Starobinsky (1985)).This model remains viable after the detection of the CMB anisotropies on large angular scales by COBE (Kofman, Gnedin & Bahcall 1993), and it is now perhaps the most promising CDM variant (Bagla, Padmanabhan & Narlikar 1995; Ostriker & Steinhardt 1995). As well known, one important motivation for a positive cosmological constant Λ is that it provides the possibility to accomodate both a high Hubble "constant", $h > 0.6$, and a sufficiently old universe, $t_0 > 11$ Gyrs. Also, the baryon fraction in clusters seems to imply $\Omega \leq 0.55$ ($\Omega = 1 - \Omega_\Lambda$ stands for the total matter density including CDM and baryons). The most recent strong argument in favour of $\Omega < 1$ (and $\Omega \simeq (0.2 - 0.4)$) follows from the evolution of rich galaxy clusters (Bahcall, Fan & Cen 1997, see also Fan, Bahcall & Cen 1997).

Up to now, these two possibilities were considered as mutually exclusive. Now we want to unite them and compare the BSI CDM model including a cosmological constant with the observational data. The reasons for this are the following. First, it can enlarge the allowed cosmological parameters window. Second, the possibility exists that the initial power spectrum of scalar (density) perturbations in the Universe is not scale free but has instead some non-trivial structure near $k = 0.05\,h$ Mpc^{-1}. In fact observations may point to such a feature: the analysis of the three-dimensional distribution of rich Abell galaxy clusters located in superclusters, performed in Einasto *et al.* (1997a), pleads for an unexpected spatial quasi-periodicity of the data (see also Einasto *et al.* 1997b, 1997c). Also, the spatial distribution of all Abell clusters of galaxies has a well-marked peak in the power spectrum at $k \simeq 0.05\,h$ Mpc^{-1} (Einasto *et al.* 1997a). The Fourier power spectrum of the spatial distribution of APM galaxies also has a feature on the same scale, though of a slightly different form (Caztanaga & Baugh, 1997). Note that the natural attempt to explain this feature by Sakharov oscillations produced by the baryon admixture to CDM does not work (Atrio-Barandela *et al.*, 1997, Eisenstein *et al.*, 1997). So this feature, if confirmed by future improved large scale structure observations, should be ascribed to the initial perturbation spectrum itself.

Therefore, we need an initial spectrum which has a non-trivial structure around some scale (preferably, with a bump) and has essentially no tilt at larger and smaller scales. The latter condition is necessary in order to have sufficiently early galaxy and quasar formation. On the other hand, this spectrum should be derivable from some first principles (e.g., it could be generated in a concrete inflationary model). Such a spectrum naturally arises in a well-defined and rather generic (though idealized, of course) inflationary model where the inflaton potential $V(\varphi)$ has a local steplike feature in the first derivative. An exact analytical expression for the scalar (density) perturbations generated in this model was found in Starobinsky (1992). It has a universal shape depending on only one parameter p. Actually, it seems to be the only example of a perturbation spectrum with the desired properties, for which a closed analytical form exists.

Thus, we suppose that the inflaton potential $V(\varphi)$ has a rapid change of slope in a neighborhood $\Delta\varphi$ of φ_0:

$$
\begin{aligned}
V(\varphi) &= V_0 + v(\varphi), && (1)\\
v(\varphi) &\simeq A_+\varphi, \quad \varphi > \varphi_0, \quad |\varphi - \varphi_0| \gg \Delta\varphi, &&\\
&\simeq A_-\varphi, \quad \varphi < \varphi_0, \quad |\varphi - \varphi_0| \gg \Delta\varphi, && (2)\\
v(\varphi_0) &= 0, \quad A_+ > 0, \quad A_- > 0. &&
\end{aligned}
$$

The resulting adiabatic perturbation spectrum is non-flat around the point $k_0 = a(t_0)H(t_0)$, t_0 being the time at which $\varphi = \varphi_0$ while $H \equiv \dot{a}/a$ is the Hubble parameter. One can show (Starobinsky 1992) that if the width $\Delta\varphi$ of the singularity is small enough, namely, $\Delta\varphi\, H(t_0)^2 \ll \min(A_+, |A_+ - A_-|)$, then the adiabatic perturbation spectrum has maximal deviation from flatness, and acquires a universal form that can be derived analytically:

$$
\begin{aligned}
k^3\Phi^2(k) \propto 1 - 3(p-1)\frac{1}{y}\left(\left(1 - \frac{1}{y^2}\right)\sin 2y + \frac{2}{y}\cos 2y\right) \\
+\frac{9}{2}(p-1)^2\frac{1}{y^2}\left(1 + \frac{1}{y^2}\right)\times \\
\left(1 + \frac{1}{y^2} + \left(1 - \frac{1}{y^2}\right)\cos 2y - \frac{2}{y}\sin 2y\right) \quad (3)\\
y = \frac{k}{k_0}, \qquad p = \frac{A_-}{A_+},
\end{aligned}
$$

where Φ is the (peculiar) gravitational potential. This expression, plotted in Fig. 1, depends (besides the overall normalization) on two parameters p and k_0. The shape of the spectrum does not depend on k_0, k_0 only determines the location of the step. For $p > 1$, the spectrum has a flat upper plateau on larger scales, even with a small bump, and a sharp decrease on smaller scales, with large oscillations though. For $p < 1$ this picture is inverted. The ratio of power between the plateaux equals p^2, and for $p = 1$ we just recover the (flat) scale-invariant Harrison-Zel'dovich spectrum. Note that this spectrum cannot be obtained in the slow-roll approximation (even with any finite number of adiabatic corrections to it). In this model it is still possible to fix freely the amount of primordial gravitational waves (GW's) for given p and normalization and we consider here the model with no GW's at all. Without the inclusion of a cosmological constant, we would be forced to consider the case $p > 1$ only, in order to increase power on large scales. Since $\Lambda > 0$ already produces a desired excess of large-scale power, we are now free to consider both cases $p > 1$ and $p < 1$.

We use for this study observational constraints both on the matter power spectrum $P(k)$ on one hand, and on the CMB anisoptropies on large, intermediate and small angular scales on the other hand, as done in Lesgourgues & Polarski (1997), and we refer the interested reader to this article for more details.

2 CONFRONTATION WITH OBSERVATION

In this work we restrict ourselves to the case of a spatially flat universe, containing cold dark matter, baryonic matter with $\Omega_B h^2 = 0.015$, and a cosmological constant Λ. Hence,

the two cosmological parameters $h = H_0/100$ and Ω_Λ are free. The present power spectrum $P(k)$ reads:

$$P(k) = \frac{4}{9}\frac{k^4}{H_0^4}\Phi^2(k)T^2(k)\Omega^{-2}(\frac{5}{3})^2\left(1-\frac{H}{a}\int_0^t a\,dt\right)^2_{t=t_0},(4)$$

where $\Phi(k)$, given by Eq. (3), is the gravitational potential at the matter dominated stage for large redshifts $z \gg 1$ when $\Omega \simeq 1$, and we put the light velocity $c = 1$. The scale factor $a(t)$ is given by the expression $a(t) = a_1(\sinh(\frac{3}{2}H_\infty t))^{2/3}$ where $H_\infty = \sqrt{\Lambda/3} = H_0\sqrt{1-\Omega_\Lambda}$, a_1 =const. The transfer function $T(k)$ is computed with the fast Boltzmann code CMBFAST by Seljak & Zaldarriaga (1996), for each value of the cosmological parameters.

The power spectrum is normalized to the four years COBE DMR data (Bennett *et al.* 1996), using $Q_{\mathrm{rms-ps}|n=1}$ (in the relevant cases, COBE scales will always correspond to the small-k flat plateau of the initial spectrum). Afterwards, we use the following tests to discriminate between each set of values of the cosmological, resp. inflationary, parameters h, Ω_Λ, resp. p, k_0):

(i) The "optical" σ_8. White, Efstathiou and Frenk (1993) give $\sigma_8 = (0.57 \pm 0.06)\ \Omega^{-0.56}$, with conservative errorbars. This is a sharp constrain at wavenumbers $k \sim 0.2\,h$ Mpc^{-1}. More recent determinations of this quantity have a tendency to decrease it to $\sigma_8 \sim 0.5$, and even a bit lower (Ebe, Cole & Frenk 1996; Viana & Liddle 1996; Ying, Mo & Börner 1997). Still, we shall use the former value (the exponent of Ω corresponds to the case of a flat Friedmann-Robertson-Walker model with a $\Lambda-$term, see below).

(ii) Peculiar velocities, deduced from the Mark III catalog, and POTENT reconstruction of the density field. In our case, the power spectrum has got strong oscillations, so we cannot simply use direct estimates of $P(k)$ at given wavenumbers (Kolatt & Dekel 1997), which would give precise constraints in the case of a smooth spectrum. We will rather use the rms bulk velocity in a sphere of radius R:

$$\langle V_R^2\rangle = \frac{f^2(\Omega)H_0^2}{2\pi^2}\int_0^\infty dkP(k)\tilde{W}_R^2(k), \qquad (5)$$

where $\tilde{W}_R(k)$ is the Fourier transform of the top-hat window function of radius R, and $f(\Omega) \equiv H^{-1}\dot{D}/D$ ($D(t)$ is the linear growth factor for inhomogeneities). Expressing f as a power law, $f(\Omega) = \Omega^r$, one can easily compute the index for a given Ω and Ω_Λ. In the interesting range $0.2 \le \Omega \le 1$, $r = 0.57 - 0.60$ for an open universe with $\Omega_\Lambda = 0$, but $r = 0.55 - 0.56$ for a flat universe with $\Omega + \Omega_\Lambda = 1$. In the following we will take $f(\Omega) = \Omega^{0.56}$. The Mark III POTENT result at $R = 50h^{-1}$Mpc (with a gaussian smoothing at $R_s = 12h^{-1}$Mpc) is $V_R = 375 \pm 85$ km s^{-1} (Kolatt & Dekel 1997). The cosmic variance (the possible dissimilarity between the rms value of $\langle V_R^2\rangle$ and the particular realization in our local neighborhood) is quite large for this quantity (~ 100 km s^{-1}), and can be added in quadrature with the previous errorbar, leading to a global uncertainty $\sigma \simeq 130$ km s^{-1}. This test is mainly sensitive to wavenumbers $0.01 \le k \le 0.06\,h$ Mpc^{-1}.

(iii) Redshift surveys. Since they are strongly bias-dependent, redshift surveys give indications about the shape of $P(k)$. Here again, due to the oscillations, instead of using some sets of estimates at given wavenumbers, one has to convolve the spectrum with the window functions of a given experiment and compare with the raw data. We use the count-in-cells analysis of large-scale clustering of the Stromlo-APM redshift survey. Taking other experiments into account would slightly improve the precision, but not change the results, since Stromlo-APM is in very good agreement with other redshift surveys, as can be seen in Peacock & Dodds (1994). After normalizing the spectrum to $\sigma_8 = 1$, we compute the variance σ_l^2 in cells of size $l\,h^{-1}$Mpc, and compare it with the data (Loveday *et al.*, 1992), consisting of nine points (assumed to be independent, with error bars treated as 2σ ones), through a χ^2 analysis. Since we can vary four parameters (plus the overall normalisation, which is irrelevant for this test), $\chi^2 \le 5$ is excellent, whereas $\chi^2 \ge 15$ is bad.

(iv) CMB anisotropies. We compute the curves $l(l+1)C_l$ using CMBFAST, and compare it with some preliminary measurements. At the moment, there are still many uncertainties, and we only have global indications on the C_l's curve. As far as the first peak is concerned, a sixth order polynomial fit to the full available data set gives $A_{peak} \equiv [l(l+1)C_l/2\pi]^{1/2} = 28 \times 10^{-6}$ with $l = 260$ (Lineweaver & Barbosa, 1997), but it is very difficult to calculate an errorbar for this quantity in the general case. CAT and OVRO give an indication on the amount of power on small scales, but do not constraint the position and height of the secondary peaks.

Since the precision of these measurements is increasing very quickly, we do not intend in this work to perform a full χ^2 analysis, using each result and the corresponding window function (to find which parameters yield the best agreement). We prefer to calculate the C_l's and comment our results in such way that in a few years one could easily update the analysis, restrict the allowed parameters window and eventually rule out the model. This is why we concentrate on the position and height of the peaks. We will use CMB data to eliminate parameters only if there is an obvious discrepancy between the predicted curve and the observations.

3 RESULTS

Starting with a flat spectrum, we explore the range $h = 0.5, 0.6, 0.7$ and $0 \le \Omega_\Lambda \le 1$. For each value of h, there are some Ω_Λ's in agreement with the σ_8 constraint: ($h = 0.5$, $0.45 \le \Omega_\Lambda \le 0.50$), ($h = 0.6$, $0.55 \le \Omega_\Lambda \le 0.60$) and ($h = 0.7$, $0.65 \le \Omega_\Lambda \le 0.70$). These windows are inside the limits $\Omega h \simeq 0.25 - 0.30$ found in Kofman, Gnedin & Bahcall (1993), and are in good agreement with other tests: bulk velocity, with $V_{50} \simeq 300$ km s^{-1}, and count-in-cells, with $3 < \chi^2 < 6$. There is no obvious contradiction with CMB measurements, and the first Doppler peak is fairly high: $A_{peak} \simeq (26 - 29) \times 10^{-6}$. Therefore, it is not necessary to depart from a flat spectrum in order to explain all observations (apart, of course, from a possible spike in the spectrum at $k \approx 0.05\,h$ Mpc^{-1}) if h and Ω_Λ turn out to be close to these values.

However, as we shall see, the steplike spectrum is compatible with a larger subset (h, Ω_Λ). It also predicts some specific features in $P(k)$ and C_l curves that could easily be observed or ruled out by future experiments, so we are *not*

just adding some extra degeneracy. More precisely, one can think of a spectrum with:

A. $p > 1$, in order to get less power on small scales in the primordial spectrum. To compensate the loss of power in $P(k)$, we will allow smaller values of the cosmological constant. This will lower the CMB anisotropies. Then, to avoid a problematic collapse of the first acoustic peak, like in double inflation (Lesgourgues & Polarski, 1997), k_0 must be chosen so that multipoles up to $l \sim 200$ (at least) are given by the upper plateau of the primordial spectrum. This means that we can forget any $k_0 < 0.03\,h$ Mpc^{-1}. For $k_0 \simeq 0.03\,h$ Mpc^{-1}, the first peak is even enhanced by a few percents by the global maximum of the primordial spectrum (the "bump" at the extremity of the upper plateau).

B. $p < 1$, in order to get more power on small scales in the primordial spectrum, and therefore allow some higher values of the cosmological constant which would be excluded by small scale constraints (for instance, σ_8) in case of a flat spectrum ($p = 1$). Since CMB peaks grow with Ω_Λ, multipoles $l \gg 2k_0/a_0H_0$ will be unusually large. A priori, in this case, the step is anywhere between the COBE and σ_8 scales: $0.003 \le k_0 \le 0.1\,h$ Mpc^{-1}. However, in this case the spectrum (3) has a well-pronounced sharp maximum at $y \simeq 3.5$ for the values $p \simeq 0.8$ which are the most interesting ones as will be seen below (for $p \ll 1$ the maximum is located at $y \simeq 3.14$). So, if we want to use this bump to explain the feature in the cluster spectrum at $k \simeq 0.05\,h$ Mpc^{-1}, k_0 should be taken $\approx 0.015\,h$ Mpc^{-1}. Note that this possibility was not expected and discussed before.

3.1 The case $p > 1$

We consider first the case $h = 0.5$. Assumption A turns out to be successfull with respect to the first three tests in many cases: for any $0 \le \Omega_\Lambda \le 0.5$, one can find a large allowed window in the (p, k_0) plane. Of course, a smaller Ω_Λ will lead to a higher range for p. For instance, when $\Omega_\Lambda = 0.3$, we find $1.3 \le p \le 1.7$ and $0.03 \le k_0 \le 0.06\,h$ Mpc^{-1}. We must stress that satisfying the three tests is a success, since most other models satisfy two of them at most. For instance, if a tilted spectrum leads to a correct σ_8 and χ^2, the bulk velocity will be generally too low, unless a very large cosmic variance is invoked. Double inflation will predict a higher V_{50}, but still under the -1σ errorbar. In the present model, V_{50} is much higher, in very good agreement with observations, as can be seen on table I (second line) for one example, because there is at least as much power on scales $0.01 \le k \le 0.06\,h$ Mpc^{-1} as for a scale invariant spectrum.

Including CMB anisotropies in the tests provides two independent constraints:

- on one hand, the position and height of the first peak are essentially related to cosmological parameters, not inflationary parameters. Indeed, as we said previoulsly, in the relevant cases ($k_0 \ge 0.03\,h$ Mpc^{-1}), the first peak is deduced from an essentially scale invariant spectrum (with only a little enhancement proportional to p, but $\le 10\%$, on A_{peak} in viable cases), and depends only on h and Ω_Λ.

The position and height of the first peak are not precisely constrained by observations at the moment, neither by Saskatoon (whose calibration is under progress: Leicht, in preparation), nor by MSAM. Inside the allowed (p, k_0) window found previously, we find $24\times10^{-6} < A_{peak} < 30\times10^{-6}$, in good agreement with current limits, so we cannot exclude any set of parameters.

- on the other hand, experiments on the secondary peaks scales constrain p and k_0. The global height of the multipoles at $400 < l < 1500$ depends on p, whereas k_0 gives the detailed shape at these scales (for instance, the ratio between the peaks), by shifting the maxima and minima of the primordial spectrum in $k-$space.

At the moment, observations do not indicate a detailed shape, but from CAT and OVRO we know that multipoles on such scales should range basically between $A_l = [l(l+1)C_l/2\pi]^{1/2} = 10 \times 10^{-6}$ and $A_l = 25 \times 10^{-6}$. Since we are not dealing with the detailed window function of each measurement, we must be extremely conservative. Using the second point of CAT, one can state that a C_l curve that would not reach $A_l = 12 \times 10^{-6}$ (the -1σ value) in the range $550 < l < 720$ (for which the window function is above half of its peak value) can be confidently excluded. The reason for which we use this particular point is that the associated window function does not interfear with the first acoustic peak: it is probing power only on the scales of the secondary ones. This restriction provides, for each value of the cosmological constant, an upper limit on p, and we find that for $0 \le \Omega_\Lambda < 0.2$, all sets of parameters are ruled out. This is an indirect constraint on Ω_Λ. For $0.2 \le \Omega_\Lambda \le 0.5$, the previously found windows still hold.

Finally, for $0.2 \le \Omega_\Lambda \le 0.5$, all the constraints can be satisfied by some values of p and k_0 in the ranges $1 \le p \le 1.7$ and $0.03 \le k_0 \le 0.07\,h$ Mpc^{-1}. To illustrate this case, we show in table I (second line) a particular example: $h = 0.5$, $\Omega_\Lambda = 0.3$, $p = 1.3$, $k_0 = 0.03\,h$ Mpc^{-1} (Ω_Λ is chosen to obtain the preferred value $A_{peak} = 28\times10^{-6}$, and p is as low as possible, in order to maximize small scales anisotropies). We also give an example of the case $\Lambda = 0$, $p = 2.1$, though it is excluded by CAT. The corresponding power spectra are plotted in Fig. 3, the CMB anisotropies in Fig. 4.

This type of model could be easily discriminated by the forthcoming improvements of redshift surveys and CMB observations. The former might state about the little well predicted in the $P(k)$ around $k \simeq (0.1 - 0.2)\,h$ Mpc^{-1}. The latter will soon indicate:

- first, the position and amplitude of the first peak, i.e., h and Ω_Λ (in the framework of this model).
- second, power on small scales, i.e. p.
- finally, the shape of secondary peaks, i.e. k_0.

This model is very unlikely to be degenerate with some other one (for instance, other cosmological parameters plus tilted spectrum) from the point of view of CMB anisotropies, because it predicts a tremendously high ratio between multipoles at scales $l \sim 200$ and $l \sim 600$ (recall that, in contrast with tilted, $n < 1$, or with double inflationary models, small scales are lowered however intermediate scales are preserved).

A similar analysis can be performed for higher h values. Since increasing h lowers the CMB multipoles, p is more restricted now by the constraints on small scales anisotropies. At h=0.6, possible models have $0.4 \le \Omega_\Lambda \le 0.6$, $1 \le p \le 1.5$

and $0.03 \le k_0 \le 0.07\,h$ Mpc^{-1}. The first peak reaches lower values as well: $26 \times 10^{-6} \le A_{peak} \le 27 \times 10^{-6}$. When $h = 0.7$, we find $0.5 \le \Omega_\Lambda \le 0.7$, $1 \le p \le 1.4$, $0.03 \le k_0 \le 0.07\ h$ Mpc^{-1} and furthermore $24.5 \times 10^{-6} \le A_{peak} \le 26 \times 10^{-6}$. The resulting allowed region in the (h, Ω_Λ) plane is plotted in Fig. 2. For a few successfull examples, we give the results of the tests in Table I.

3.2 The case $p < 1$

Again, we first consider the case $h = 0.5$. When $0.45 \le \Omega_\Lambda \le 0.7$, one can find some (p, k_0) in good agreement with σ_8, V_{50} and χ^2. For instance, when $\Omega_\Lambda = 0.6$, the allowed region is $0.75 \le p \le 1$ and $0.003 \le k_0 \le 0.04\,h$ Mpc^{-1}. In the last subsection, it was found that for $p > 1$, the most compelling constraint was on σ_8. Now the three tests play an important part in the definition of the allowed region. Indeed, the above mentioned sharp maximum appears in $P(k)$, preceeded at larger scales by a depression at $y \approx 1.2$ (the inverted bump of the case $p > 1$). As a result, the power spectrum $P(k)$ has no pronounced second maximum at the place where it exists for $p = 1$, namely $k \simeq 0.05\,\Omega^{-1} h$ Mpc^{-1} . Note that this depression would become very pronounced in the case $p \ll 1$, its position in this limit being given by $y = \sqrt{2.5p}$ (Starobinsky, 1992). When $k_0 > 0.015\,h$ Mpc^{-1}, the bulk velocity is sometimes too small due to this little depression. On the contrary, when $k_0 < 0.015\,h$ Mpc^{-1}, the maximum often generates excessive bulk velocities.

As expected, the CMB anisotropies are amplified by both the comological constant and the primordial spectrum step. The basic picture is that the C_l's are enhanced by a factor p^2 for $l > 2k_0/a_0H_0 \simeq 12\,000k_0$. When $k_0 = 0.01 - 0.02\,h$ Mpc^{-1}, the first peak is enhanced by the maximum of the primordial spectrum, so its location and maximum value are highly dependent on all parameters, including k_0 and p (in contrast with the case $p < 1$). The secondary peaks are given by an almost flat region of the primordial spectrum, so they depend on all parameters, k_0 excepted.

As in the previous subsection, we can use the last CAT point to reduce the allowed window, confidently excluding any C_l curve that would not pass $A_l = 21 \times 10^{-6}$ (the -1σ value) in the range $550 < l < 720$. This rules out many low p values for a given Ω_Λ. In fact models with $\Omega_\Lambda > 0.5$ do not survive. At $\Omega_\Lambda = 0.5$ we find the allowed window: $0.85 \le p \le 1$, $0.003 < k_0 < 0.04\,h$ Mpc^{-1}. Similarily, when $h = 0.6$, successfull models can be found for $0.55 \le \Omega_\Lambda \le 0.65$, extending the validity range of the scale invariant model. At $\Omega_\Lambda = 0.65$ the allowed window is $0.80 < p < 0.85$, $0.003 \le k_0 \le 0.04\,h$ Mpc^{-1}. Finally, when $h = 0.7$, $0.65 \le \Omega_\Lambda \le 0.75$ is allowed. At $\Omega_\Lambda = 0.72$ we find $0.80 < p < 0.85$, $0.003 \le k_0 \le 0.04\,h$ Mpc^{-1}. These results are also summarized in Fig. 2. Table I contains a few examples, and for one of them the power spectrum and CMB anisotropies are illustrated in Fig. 3 and Fig. 4.

At first sight, the case $p < 1$ is not interesting since it does not extend very much the allowed region for (h, Ω_Λ): good results are obtained for cosmological parameters that are not in conflict with the scale invariant model. The interest of the $p < 1$ steplike spectrum lies in the prediction of specific features, namely:

- a sharp maximum in $P(k)$. The steplike model with $k_0 \simeq 0.015\,h$ Mpc^{-1} and $p \simeq 0.8$ could perfectly explain the form of the cluster spectrum with a peak at $k_0 \simeq 0.05\,h$ Mpc^{-1} (see Fig. 5).
- large CMB anisotropies. The Saskatoon experiment (Netterfield *et al.* 1997) indicates a too high first peak that cannot be explained by current flat CDM models (unless $h = 0.2 - 0.3$ is allowed). These measurements might be contaminated by some systematic effects, as indicated by MSAM third flight result (Cheng 1997). However, if the Saskatoon points are confirmed, the flat Λ+CDM steplike model with $p < 1$ would be a good condidate, since it predicts high anisotropies, without getting in conflict with constraints on $P(k)$, and without requiring $h < 0.5$. For instance, when $p = 0.85$, the C_l values increase by 40% at the scales of secondary peaks, and even of the first peak if $k_0 \le 0.01\,h$ Mpc^{-1}. When $h = 0.6$, $\Omega_\Lambda = 0.65$, $p = 0.85$, and $k_0 = 0.01h$ Mpc^{-1}, we find $A_{peak} = 35 \times 10^{-6}$.

4 CONCLUSIONS AND DISCUSSION

We have compared the CDM+Λ cosmological model with a BSI initial spectrum of adiabatic perturbations given by Eq. (3) with recent observational data. The model is determined by four fundamental parameters Ω_Λ, A_+, $A_- \equiv pA_+$ and k_0, (in addition to the Hubble constant H_0) out of which one (A_-) is fixed by the normalization to the COBE data. The number of observational tests we use is sufficient to rule out many primordial spectra well-motivated by inflationary theories. For instance, to enlarge the allowed (h, Ω_Λ) window, one could think of introducing a tilted or double inflationary spectrum to reconcile observations on large scales (COBE) and small scales (σ_8). However, there will be a generic lack of power on intermediate scales (bulk velocity, first CMB peak). Moreover, exactly at these scales, there may be an unexpected excess of power. The initial spectrum that we study here allows a significant enlargement of the allowed (h, Ω_Λ) region, especially smaller Ω_Λ's, without supressing power at intermediate scales.

We have found allowed regions in the (h, Ω_Λ) parameter plane for p lying in the region $(0.8 - 1.7)$. These allowed regions are larger than in the case of a flat initial spectrum ($p = 1$). The most interesting, and alltogether unexpected, successfull model appears to be that with an inverted step $p < 1$, where the power at intermediate scales is even more enhanced. It appears that this latter case is suitable for the description of the feature in the cluster spectrum found in Einasto *et al.* (1997a, 1997b, 1997c). The most distinctive feature of the class of models in question is the suppression of the second and higher acoustic (Doppler) peaks in the case $p > 1$, and their enhancement in the opposite case. That is why the CAT CMB experiment appears the most restrictive for the model. So, the exact measurement of C_l for $l \sim 500$, i.e. around the second acoustic (Doppler) peak, will be the crucial test for this model. The forthcoming improvements of CMB anisotropies measurements, especially baloon and satellite experiments, should be able either to rule out this model or to detect its signature in the next ten years.

On the other hand, the increase of the allowed region in the (h, Ω_Λ) plane and the allowed range for p itself are not large. This shows the remarkable robustness of the CDM+Λ cosmological model with the simplest inflationary

Table 1. Results of the tests for the different models. For each value of h, we show the best model with a flat spectrum ($p = 1$), a step towards large scales ($p > 1$), and a step towards small scales ($p < 1$). For $h = 0.5$ we also give the best model with $\Omega_\Lambda = 0$.

h	Ω_Λ	p	k_0 (h Mpc^{-1})	$\Omega^{0.56}\sigma_8$	V_{50} (km s^{-1})	χ^2	first peak l_{peak}, A_{peak}	second peak l_{peak}, A_{peak}
0.5	0	2.1	0.030	0.63	390	7.6	215, 26×10^{-6}	475, 10×10^{-6}
	0.3	1.3	0.030	0.63	345	6.2	225, 28×10^{-6}	515, 16×10^{-6}
	0.5	1	(flat)	0.54	300	3.7	235, 29×10^{-6}	555, 22×10^{-6}
	0.5	0.85	0.015	0.63	310	3.1	260, 32×10^{-6}	555, 25×10^{-6}
0.6	0.55	1.2	0.030	0.53	330	4.6	220, 27×10^{-6}	510, 18×10^{-6}
	0.60	1	(flat)	0.54	305	3.5	225, 27×10^{-6}	530, 21×10^{-6}
	0.65	0.8	0.015	0.57	300	2.6	255, 31×10^{-6}	530, 27×10^{-6}
0.7	0.60	1.2	0.030	0.57	340	4.7	210, 25×10^{-6}	485, 18×10^{-6}
	0.65	1	(flat)	0.57	315	4.1	215, 26×10^{-6}	500, 20×10^{-6}
	0.70	0.8	0.015	0.59	310	2.7	240, 28×10^{-6}	505, 26×10^{-6}

initial conditions ($\Omega_{\rm tot} = \Omega + \Omega_\Lambda = 1$; $n \simeq 1$). Also, the fact that the allowed values of p are close to unity indicates that the form of the inflaton potential $V(\varphi)$ is close to the case of a discontinuity in its second, not first derivative, which is more natural since, e.g., it can occur as a result of an equilibrium second-order phase transition (some kind of non-analytic behaviour of $V(\varphi)$ is required in any case to obtain significant deviations from the flat perturbation spectrum). Consideration of the latter case is under progress.

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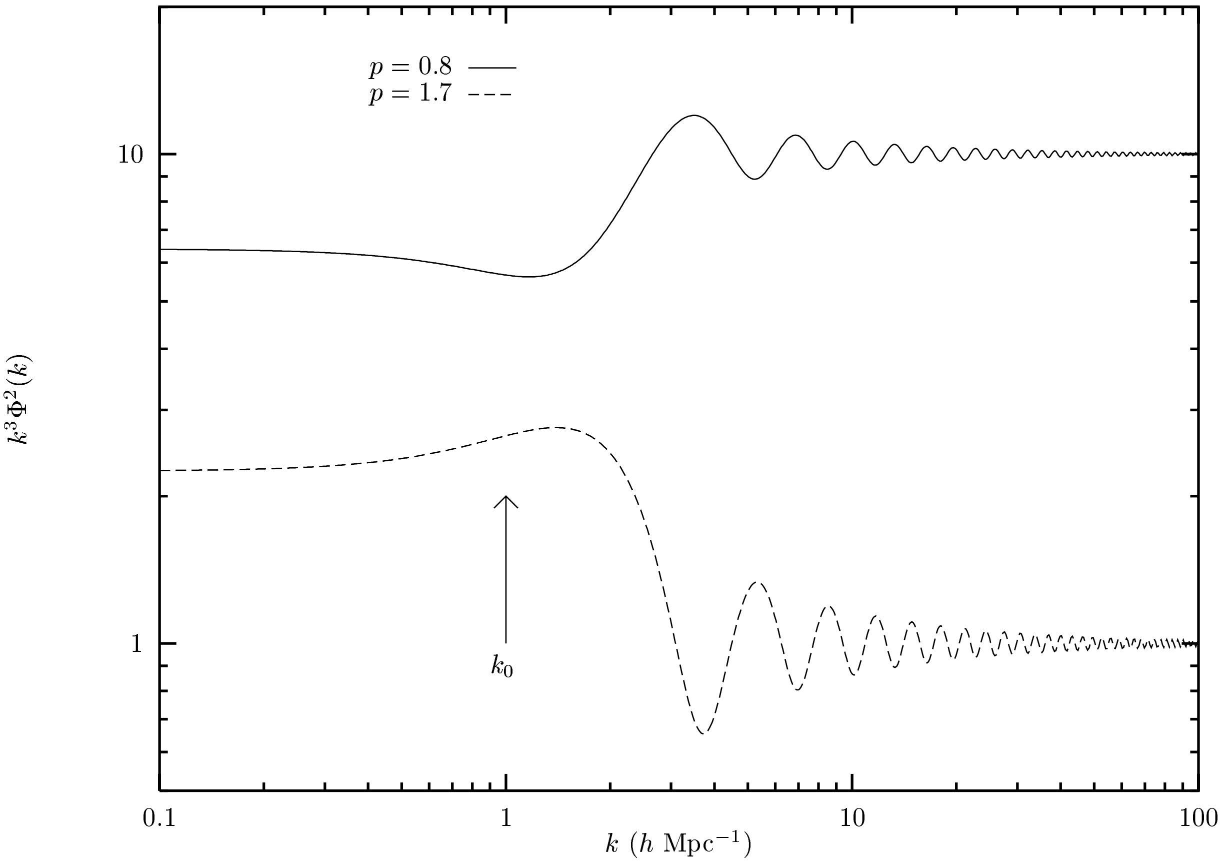
Figure 1. Primordial power spectrum for ($p = 0.8$, $k_0 = 1\,h$ Mpc^{-1}) and ($p = 1.7$, $k_0 = 1\,h$ Mpc^{-1}). At this stage, the normalisation of each spectrum is arbitrary. It is important to note that in both cases, the maximum is located at the extremity of the upper plateau.

Figure 2. Allowed region in the cosmological parameters plane (h, Ω_Λ). The lower hatched region corresponds to models with $p \geq 1$, the upper one to models with $p \leq 1$. Inside the intersection, a scale-invariant spectrum ($p = 1$) is allowed. The steplike model is seen to enlarge significantly the allowed region.

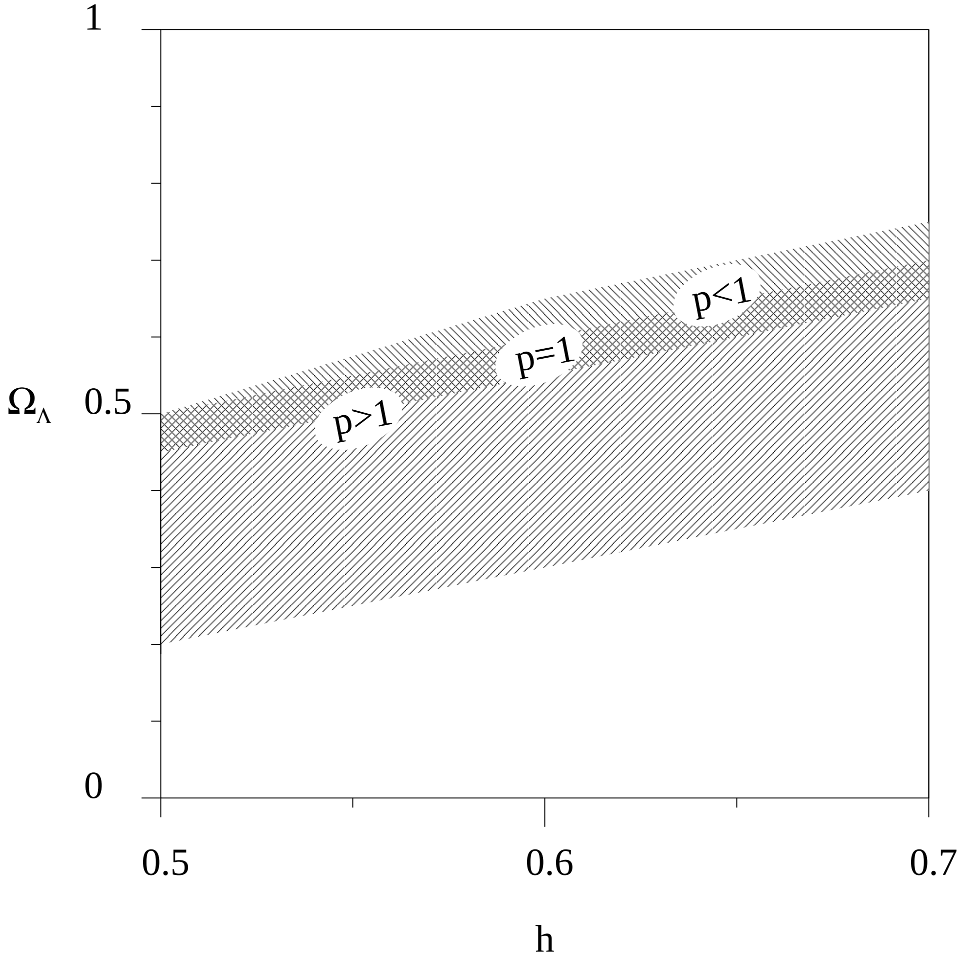
Figure 3. Power spectrum for a few models from Table 1 : a model with $\Omega_\Lambda = 0$ (in conflict with CAT), and three viable models with $p > 1$, $p = 1$ and $p < 1$.

Figure 4. CMB anisotropies for the same models as in the previous figure. We also plot a few measurements, including Saskatoon (Netterfield 1997) recalibration (Leicht, in preparation) and new preliminary CAT (Baker, in preparation) and OVRO (Leicht, in preparation) results. We have in order of appearance for growing l: COBE (3 points), Tenerife, South Pole, Saskatoon (5 points), MAX (2 points), MSAM, CAT (2 points) and OVRO.

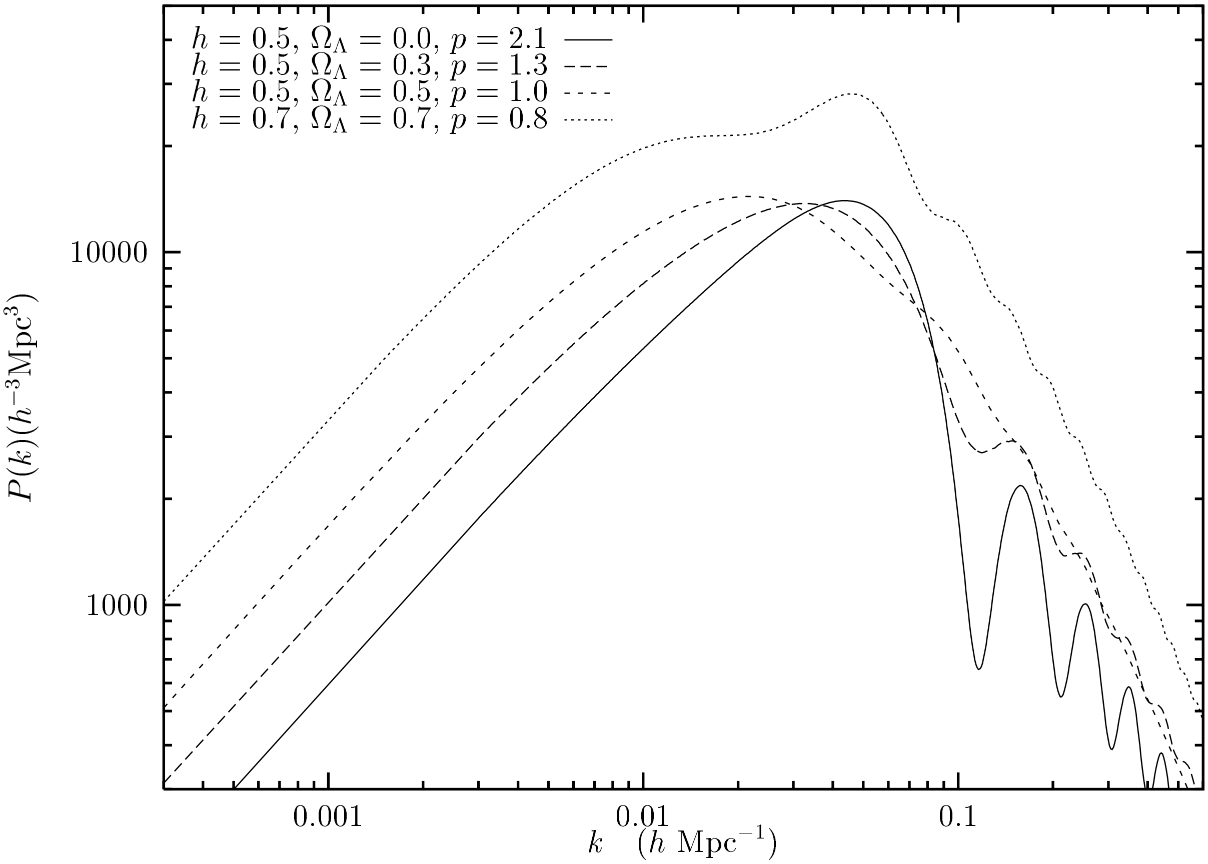
Figure 5. Theoretical power spectrum for $h = 0.7$, $\Omega_\Lambda = 0.72$, $p = 0.75$ ans $k_0 = 0.016\,h\,\mathrm{Mpc}^{-1}$ compared with the power spectrum of rich Abell galaxy clusters, taken from Einasto *et al.* (1997a) and divided by $b^2 = 5$.



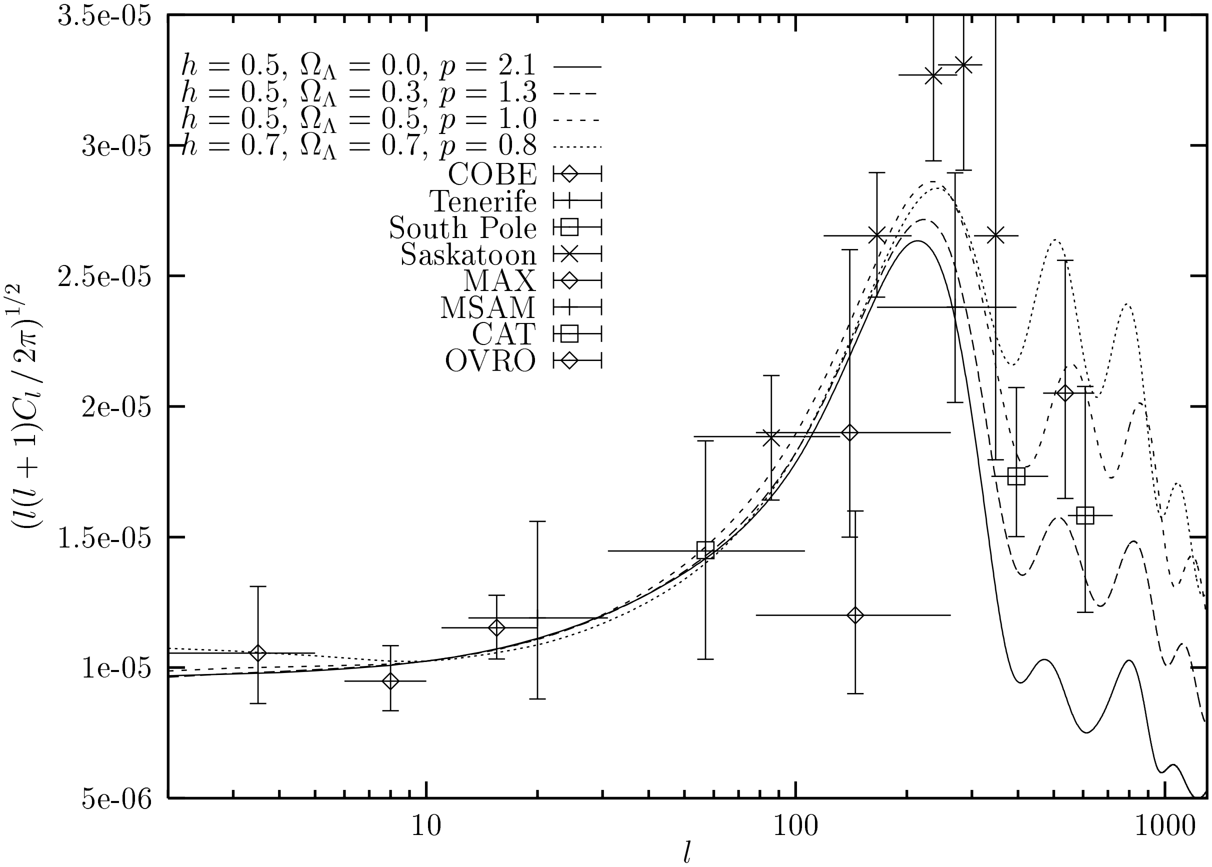
$p = 0.8$
$p = 1.7$
10
1
$k^3\Phi^2(k)$
k_0
0.1
1
10
100
k (h Mpc^{-1})



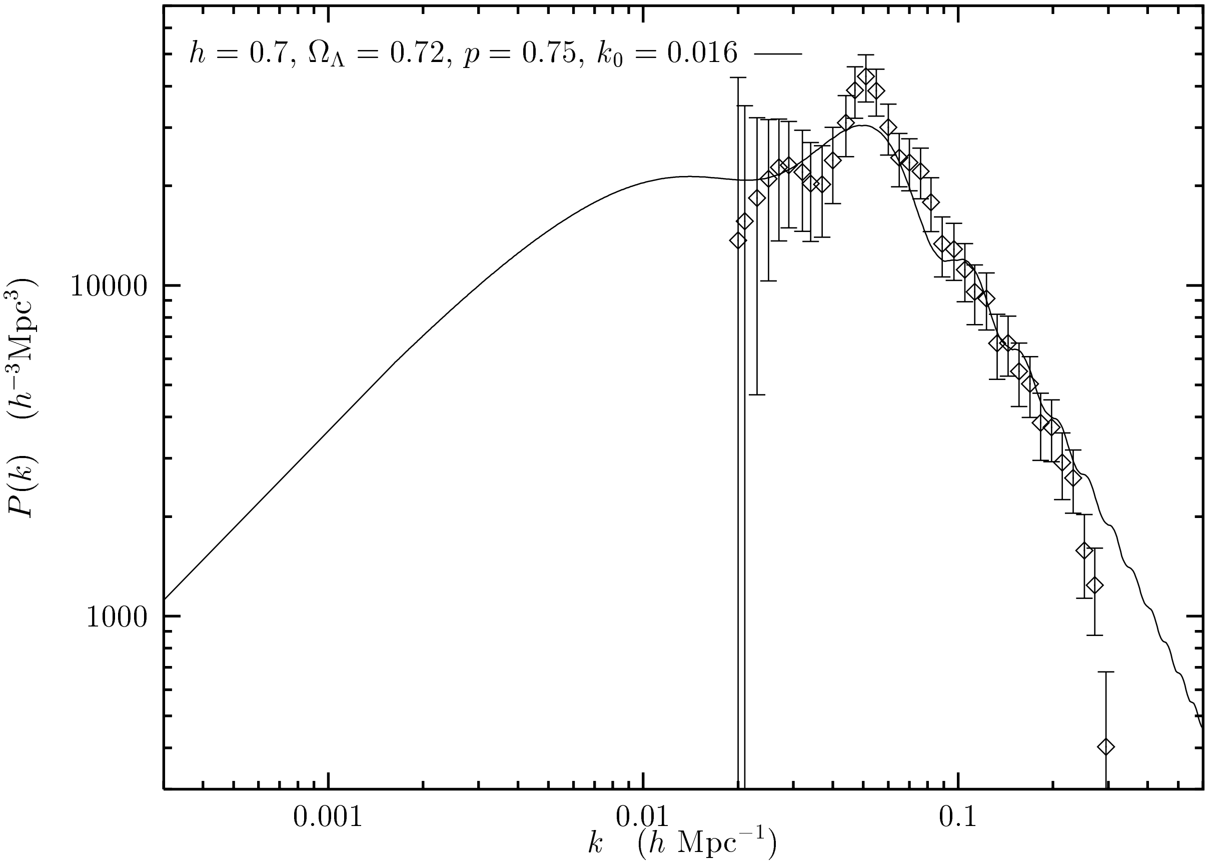
1
0.5
0
Ω_Λ
p<1
p=1
p>1
0.5
0.6
0.7
h



$h = 0.5$, $\Omega_\Lambda = 0.0$, $p = 2.1$
$h = 0.5$, $\Omega_\Lambda = 0.3$, $p = 1.3$
$h = 0.5$, $\Omega_\Lambda = 0.5$, $p = 1.0$
$h = 0.7$, $\Omega_\Lambda = 0.7$, $p = 0.8$
10000
1000
$P(k)(h^{-3}\mathrm{Mpc}^3)$
0.001
0.01
0.1
$k \quad (h\ \mathrm{Mpc}^{-1})$



$h = 0.5$, $\Omega_\Lambda = 0.0$, $p = 2.1$
$h = 0.5$, $\Omega_\Lambda = 0.3$, $p = 1.3$
$h = 0.5$, $\Omega_\Lambda = 0.5$, $p = 1.0$
$h = 0.7$, $\Omega_\Lambda = 0.7$, $p = 0.8$
COBE
Tenerife
South Pole
Saskatoon
MAX
MSAM
CAT
OVRO
$(l(l+1)C_l\,/\,2\pi)^{1/2}$
3.5e-05
3e-05
2.5e-05
2e-05
1.5e-05
1e-05
5e-06
10
100
1000
l



$h = 0.7$, $\Omega_\Lambda = 0.72$, $p = 0.75$, $k_0 = 0.016$
10000
1000
$P(k) \quad (h^{-3}\mathrm{Mpc}^3)$
0.001
0.01
0.1
$k \quad (h\ \mathrm{Mpc}^{-1})$